M332 G

THE OPEN UNIVERSITY

Mathematics: A Third Level Course

A GUIDE TO THE COURSE: COMPLEX ANALYSIS

Prepared by the Course Team

Please read this before reading any other M332 course material.

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M332 is a half-credit course of fifteen units, two of which are NO TEXT. The titles of the units are:

0 Real Analysis Review
1 Complex Numbers
2 Continuous Functions
3 Differentiation
4 Integration
5 Cauchy's Theorem I
6 Taylor Series
7 NO TEXT
8 Singularities
9 Cauchy's Theorem II
10 The Calculus of Residues
11 Analytic Functions
12 NO TEXT
13 Properties of Analytic Functions
14 Laplace Transforms

(Prior to 1982 the course contained a *Unit 15* and a *Unit 16*, but no *Unit 0*.)

You need study only one unit (of your choice) out of the last two units. You will automatically be sent *Units 13* and *14* in plenty of time for you to look through them and decide which interests you most. The last Tutor-marked Assignment (TMA) and the Examination are designed to cope with whichever you choose.

Quite often units are bound together. At the front of each bound volume you will find a list of the contents of all the units it contains, and at the back you will find an index and glossary of notation for these units.

The course has no set book since all you will need can be found in the units. The only references to other texts either provide further reading (on non-examinable material) for the interested student, or remind you of what you should recall from other courses.

Optional Material

Minor changes are made to the content of the course from time to time, in the light of experience. You should therefore consult the Stop Presses to check which sections (if any) of the units are not part of the course in the current academic year.

SAQs and Problems

Part of the teaching is done through self-assessment questions (SAQs) and problems. We think that you should try to solve these questions and problems (*on your own*, before reading their solutions!) as you work through the units. The solutions immediately follow the questions.

Unit 0

M332 was originally written as a sequel to *M231, Analysis*. This latter course has now been replaced by *M203, Introduction to Pure Mathematics*. These two courses adopt a superficially different approach, one using the "ε-δ approach" and the other the "nests approach". *Unit 0* is intended as a revision of some of real analysis for those students who have studied the ε-δ approach in *M231* or elsewhere, and as general revision plus an extended version of *M203, Unit 30, Section 30.5*, for those students who first studied real analysis via the nests approach in *M203*.

Television and Radio

The course units are supplemented by 11 television programmes and 7 radio programmes.

The television programmes are intended to help you understand the more difficult parts of the units. In the radio programmes we try to show you how to solve the sorts of mathematical problems that arise in the course. For your convenience, the radio programmes are not on "live" radio, but are sent to you on two audio-cassettes.

Before each television or radio programme, you should read the introduction to the appropriate Broadcast Notes. These will help tell you what we feel you ought to know in order to get the fullest benefit from the programme. You should have the appropriate Broadcast Notes with you during the broadcast. The main part of each set of notes summarizes the main points of the programme, and you should not need to take your own notes. Following this summary are a few short self-assessment questions together with their solutions. These are intended to test your immediate recollection of the important points of the programme, and you should aim to do them immediately after the broadcast.

All the Broadcast Notes are bound in a single volume, except those for TV0.

Radio Programme 8 and Television Programme 10 no longer form part of the course, and so are no longer available to you.

Supplementary Materials

In each mailing to you there should be a Contents Checklist, listing all the items included in that mailing. If you do not receive a checklist, or your mailing does not include all the items on the checklist, please notify

The Manager
Correspondence Services
The Open University
PO Box 50
Walton Hall
Milton Keynes MK7 6AE

so that we can make amends.

Stop Presses

These contain important information and should be read *before* looking at the rest of the material in the mailing. They may include lists of errata and comments on the TMA questions.

Assignment Questions

These will be bound together in blocks corresponding to the units with which they are sent. They will include explanations of which questions cover which units, as well as the usual reminder of cut-off date for receipt of your assignment by your tutor.

There are 5 TMAs, but *no* Computer-marked Assignments (CMAs). The questions for each unit should take you about 1½ hours to complete; they will not necessarily be of equal difficulty or carry equal marks. Do not worry if you cannot do all the TMA questions; you will not need full marks on a TMA to obtain a good average grade on it.

All assignment questions will contribute towards your TMA grade; there are *no* non-assessed questions.

Tutors and Tutorials

We hope that you will be able to attend the tutorials arranged at some Study Centres, but realize that this may not be possible. In any case, you can always obtain help from your tutor by writing to him or her, or by telephoning; do *not* feel shy about this!

When submitting your TMA, help your tutor to help you! Show your tutor *all* your working; do not throw away your working unless you have replaced it with something better. Do not be afraid to submit scrappy part solutions or wrong attempts; they will help your tutor find out your thoughts on the problem and your difficulties with the course. You will probably find that you will pick up marks by this process that would otherwise have been consigned to your wastebin. *And* your tutor's written comments and suggestions will help you over your difficulties.

As *M332* is a third-level course, you will meet a fairly high level of sophistication in the proofs; and we therefore expect the same level in your proofs. This is of particular importance in assignment questions, when you must check (and show that you have checked) that the hypotheses of a result are satisfied before invoking that result.

However, do not set too high a standard for yourself. At this level, mathematics is by no means trivial, and you cannot hope to understand absolutely everything. Most students at conventional universities who do a course like this find difficulties in it. By the way, you will probably find that your difficulties with the early units will be largely overcome by your study of later material!

Summer School

This course has no summer school.

Revision Materials

Among the various course materials you will receive the following:

(i) a Revision Booklet, covering *Units 1–11*;

(ii) a Guide to the Examination, including a list of those definitions and theorems that we shall want you to remember for the examination;

(iii) a Specimen Examination Paper, giving you an idea of the style and standard of difficulty of the paper;

(iv) a set of Solutions and Mark Scheme for the Specimen Paper, indicating the style of solutions expected by the Examiners, and illustrating how marks can be gained even by partial or incorrect solutions to a question.

Background Knowledge

In order to succeed in *M332* it is essential that you have studied Analysis (that is, a *rigorous* approach to Calculus) previously in *M231*, or *M212* or *M203* — *or* possess some comparable background in Real Analysis. It is not possible to do this course successfully otherwise.

M332 is more-or-less self-contained, as it explains all its notation when you first meet it. Occasionally, you will be referred to *M231* or to *Spivak* (see below) for results on real analysis which will be assumed in *M332*; in either case, we suggest that you refer to *Spivak*.

Further Reading

For results in Real Analysis to which you may wish to refer, we recommend

M. Spivak, *Calculus* (W. A. Benjamin/Addison-Wesley, 1973) paperback edition.

If you would like to see many worked problems in Complex Analysis, we recommend

M. R. Spiegel, *Complex Variables*, Schaum Outline Series
(McGraw-Hill, New York, 1964).

One of the classic texts in Complex Analysis is

L. V. Ahlfors, *Complex Analysis* (McGraw-Hill, New York)
second edition (1966), or third edition.

It starts at the beginning, but goes very far and at a very high level of sophistication, so it is not easy reading! But it gives you some idea of what lies in Complex Analysis beyond the limits of *M332*.

A fairly readable book with, however, not a great deal of motivation for results, and few worked problems is

E. C. Titchmarsh, *Theory of Functions* (Oxford University Press, 1939)
second edition.

This is not always quite as careful about rigour as *M332*; but, like Ahlfors, it is on every professional Complex Analyst's bookshelf!

M332 Course Team

The *M332* Course Team comprised Dr G. A. Read (Chairman); Dr P. D. Bacsich, Dr M. Crampin, Mr N. W. Gowar and Dr R. J. Wilson (unit authors); Mr R. J. Knight (editor); Mr D. Saunders (BBC).

The Course Team was assisted by Mr D. Goldrei, Mr T. Lister, Mr. R. J. Margolis, Dr A. R. Meetham, Mr J. E. Phythian, Mrs P. M. Shepheard Rogers, Dr C. A. Rowley and Mr D. R. Sargent; and by Mr R. Clamp, Mr H. Hoggan and Mr J. Richmond of the BBC.

Recent modifications to the course were made by Professor D. A. Brannan.

The Greek Alphabet

A	α	Alpha
B	β	Beta
Γ	γ	Gamma
Δ	δ	Delta
E	ε	Epsilon
Z	ζ	Zeta
H	η	Eta
Θ	θ (or ϑ)	Theta
I	ι	Iota
K	κ	Kappa
Λ	λ	Lambda
M	μ	Mu
N	ν	Nu
Ξ	ξ	Xi
O	o	Omicron
Π	π	Pi
P	ρ	Rho
Σ	σ	Sigma
T	τ	Tau
Υ	υ	Upsilon
Φ	ϕ (or φ)	Phi
X	χ	Chi
Ψ	ψ	Psi
Ω	ω	Omega

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